Impedance and resonance issues for a long rectangular slot in a coaxial liner

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Beam pipes in high-energy superconducting colliders require a shielding tube (liner) with pumping slots to screen cold chamber walls from synchrotron radiation. Earlier we developed a general analysis, based on a variational formulation, which includes both the realistic coaxial structure of the beam pipe with a liner and the effect of finite wavelength in the calculation of the coupling impedance of a rectangular slot in a liner wall of zero thickness. In the present paper we use this analysis to study the frequency dependence of the coupling impedance of a longitudinal rectangular slot, which is of great interest as a shape of pumping slot. Resonant effects in the coupling impedance involving the ratio of the slot length to the wavelength are explored. We also present the analytic results for the real part of the impedance at low frequencies for a liner wall of both negligible and finite thickness. [S1063-651X(97)09412-9]

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I. INTRODUCTION

The pumping slots in the liner are the chamber discontinuities, and electromagnetic fields diffracted by them can affect beam stability. This beam-chamber interaction can be described in terms of the coupling impedance. We consider an azimuthally asymmetric single rectangular slot in the inner conductor of a coaxial liner, whose thickness we take to be zero.

Since the driving current on axis is proportional to $\exp(-jkz)$, the problem is simplified by obtaining results for an even driving current $\cos kz$ and an odd driving current $-j\ \sin kz$ separately. This separation is needed to construct a variational form for the impedance.

Formulas for direct numerical computation of the impedance of a rectangular slot have been obtained earlier [1], where the even and the odd parts of the impedance are calculated separately before adding them. These formulas were used to study the coupling impedance of transverse rectangular slots of different azimuthal length and the impedance of a small square hole [1]. In this paper we use this analysis [1] to study the coupling impedance of a longitudinal rectangular slot, including the possibility of resonant behavior in the slot region. We also present the analytic results for the real part of the impedance at low frequencies and discuss the effect of the wall thickness.

We denote the radius of the inner conductor (liner) by a and the radius of the outer conductor (beam pipe) by b. We then consider a rectangular slot of width w and length l in the liner wall of zero thickness.

II. IMPEDANCE OF A LONGITUDINAL SLOT

A. Validity of the static approximation for low frequencies

In the well-known Bethe small-hole approximation the imaginary part of the impedance below cutoff is given by

$$Z(\omega)=jZ_0\,\frac{\omega}{c}\,\frac{\psi-\chi}{8\pi^2a^2}, \tag{2.1}$$

with $Z_0=120\pi$ (Ω), a the inner pipe radius, c the velocity of light, ω the angular frequency, and ψ and χ the magnetic susceptibility and electric polarizability, respectively. Note that in some of the literature, where the radiation into one half space is considered, the electric polarizability and magnetic susceptibilities are redefined in terms of the effective polarizabilities $\alpha_e=-\chi/2$ and $\alpha_m=\psi/2$.

In contrast to the transverse slot studied in [1], the longitudinal rectangular slot disturbs the beam-induced current only near its ends. We therefore expect the impedance of such a slot to be determined mostly by the width and the shape of the ends. The static approximation for the imaginary part of the impedance of a long rectangular slot

$$\psi-\chi=0.3628w^3 \tag{2.2}$$

suggests that the impedance becomes independent of the slot length when the length of the slot is much larger than its width. The above formula follows from the static polynomial expressions for ψ and χ of a rectangular slot in the limit of $w/l\to 0$ [2,3]. Recently, the frequency correction for the impedance of the longitudinal elliptical slot was obtained analytically [4]. The frequency correction term indicates that the imaginary part of the impedance of a long elliptical slot is strongly reduced at finite frequencies. It is important to understand whether a similar effect is present for a rectangular slot at finite frequencies.

Numerical study of this question for a long rectangular slot shows that there is no significant reduction in the imaginary part of the impedance at low frequencies. This suggests that an elongated ellipse is somewhat different from a long rectangular slot since the current pattern will be distorted all along the elliptical slot. Therefore, the available static approximations for ψ and χ [2,3] give a reasonably good description for low frequencies ($ka<1$). Note that available polynomials for ψ and χ are accurate to 1% only in the ‘‘ideal’’ situation of a slot in a plane metallic wall without coaxial structure, i.e., when conditions $a/w\to\infty$ and $b/a\to\infty$ are valid. This is usually not the case in a real design. For example, in our numerical calculations we consider $w/a=0.25$ and finite b/a by solving the exact problem of a coaxial pipe with transverse curvature. The exact numerical

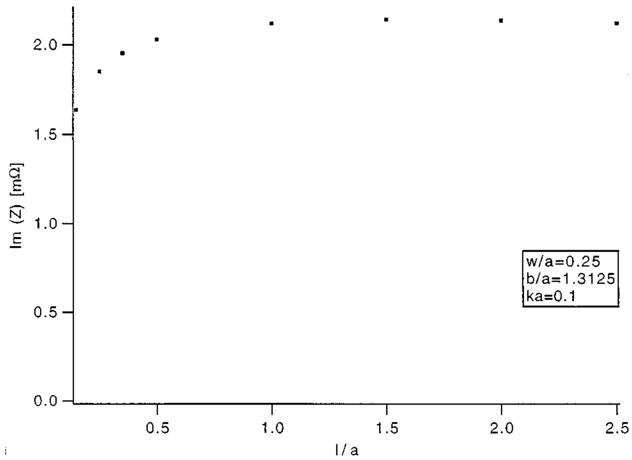

FIG. 1. Imaginary part of the impedance at frequency $ka=0.1$.

results for ψ and χ can differ from the ideal one by as much as 10–15 %, depending on the values of b/a and a/w considered. The detailed description of the accuracy issues of polynomials for ψ and χ can be found in [3]. In general, an accuracy of 10% is sufficient and therefore the available static approximation for ψ and χ can be used even for finite a/w and b/a, which are usually considered. As can be seen from Figs. 1 and 2, for the frequency $ka=0.1$ the static approximation gives reasonably good results even when the slot length is larger than the radius of the pipe. Note that reduction of the impedance for $l/a>1.5$ is very small and cannot be easily observed in the figures. For higher frequencies ($ka\geqslant 1$) the static approximation loses its accuracy at much smaller values of l/a. For example, at frequencies $ka>1$ the decrease in the imaginary part of the impedance starts when the length of the slot is less than the radius a, as can be seen from Fig. 3.

However, even for high frequencies the reduction of the imaginary part of the impedance due to the slot length is insignificant. In addition, we can see the resonant behavior of the real part of the impedance in Fig. 4.

B. Resonances

There are two potential sources of impedance resonances due to slots at high frequencies related to the distribution and

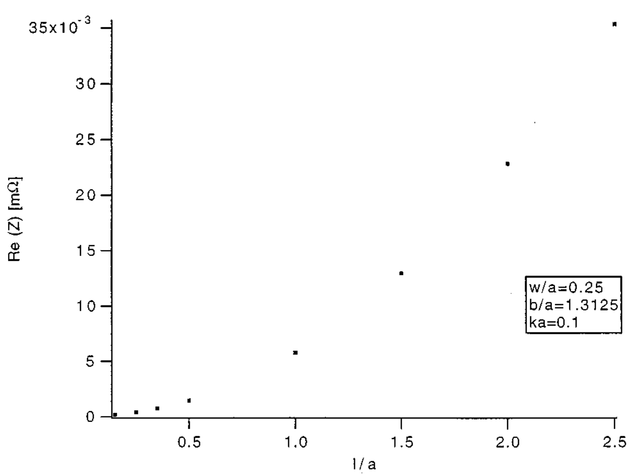

FIG. 2. Real part of the impedance at frequency $ka=0.1$.

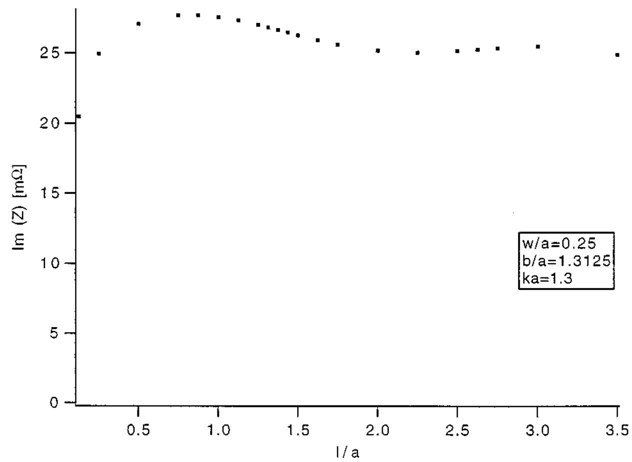

FIG. 3. Imaginary part of the impedance at frequency $ka=1.3$.

length of the slots. Resonances related to the periodicity of the slot distribution along the liner were studied by Kurennoy [5] and Gluckstern [6]. Even small random longitudinal displacements of slots from their positions in an exactly periodic array reduce the resonances greatly.

Resonances related to the length of the slot were studied in this paper. Due to the independence of the imaginary part of the impedance on the length of the slot for very long slots in the static approximation, the general tendency would be to design very long slots. However, increasing the length of the slot can bring resonances due to the slot length to low frequencies. Note that the imaginary part of the impedance is of primary concern since it is much larger than the real part. A numerical study of the resonant behavior for slots of different length was performed.

One can see the appearance of resonant behavior for the real part of the impedance in Fig. 5 for a very long rectangular slot with $l/a=5$. We identify the resonances for the imaginary part as the place where the impedance changes its sign. For the imaginary part there appears to be resonant behavior for the odd and even parts of the impedance separately, but due to the strong cancellation between the even and odd parts (for example, in static theory the electric polarizability of a very long rectangular slot exactly equals the

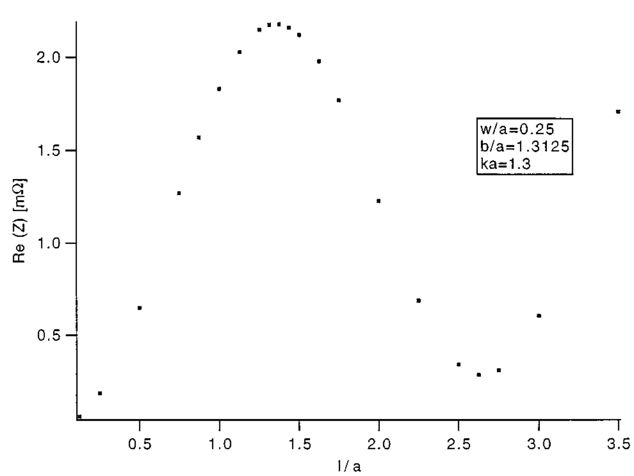

FIG. 4. Real part of the impedance at frequency $ka=1.3$.

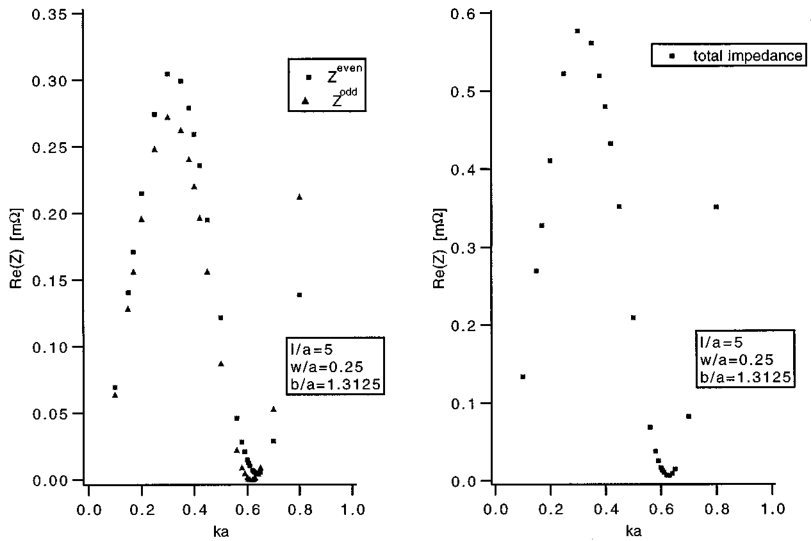

FIG. 5. Real part of the impedance of a long rectangular slot ($l/a=5$).

transverse magnetic susceptibility of such slot) no resonant behavior is seen for the total impedance (Fig. 6). For a relatively short slot the resonances occur at different frequencies for the even and odd parts of the impedance, as can be seen in Fig. 7. When we increase the length of the slot, the places where the even and odd parts change sign approach one another. First two resonances can be seen Fig. 7 for slot lengths $l/a=3$ and $l/a=5$. For a long slot the resonances occur near $kl=\pi$, which corresponds to the wavelength $\lambda=2l$. The frequency behavior of the real and imaginary parts of the total impedance for a very long rectangular slot with $l/a=5$ is shown in Fig. 8 and 9, respectively.

III. REAL PART OF THE IMPEDANCE AT LOW FREQUENCIES

Based on our earlier analysis [1], we can examine the low-frequency behavior analytically. The simple consequence of the developed theory is the analytic result for the real part of the impedance for frequencies below all possible cutoffs, including only the TEM mode [1]:

$$\mathrm{Re}\left(\frac{Z}{Z_0}\right)=\frac{k^2}{64\pi^3 a^4 \ln(b/a)}\,(\psi^2+\chi^2), \tag{3.1}$$

where $k=\omega/c$. Note that for the case of a liner wall of negligible thickness the ''inside'' susceptibility and polarizability are equal to the ''outside'' susceptibility and polarizability and we do not use any subscripts for ψ and χ. The situation is different for finite wall thickness, as will be discussed in Sec. III C.

A. Beam power losses

The power per unit length of the beam chamber dissipated due to beam fields scattered by the slots is

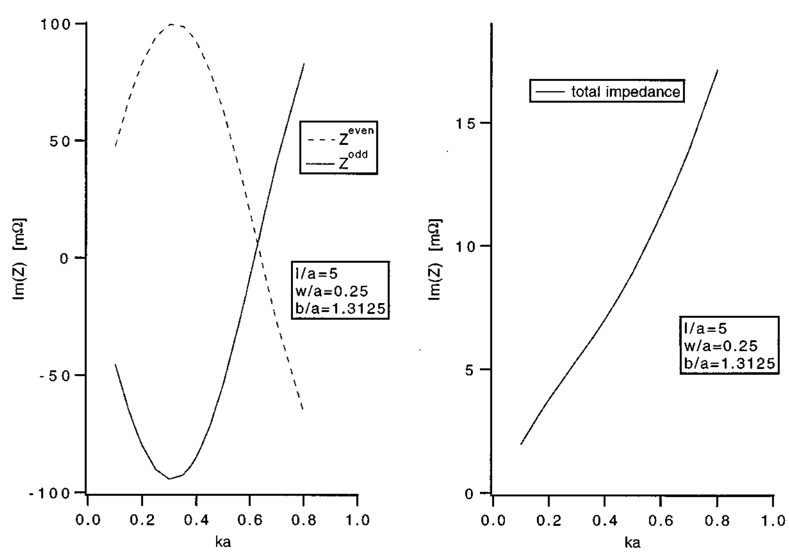

FIG. 6. Imaginary part of the impedance of a long rectangular slot ($l/a=5$).

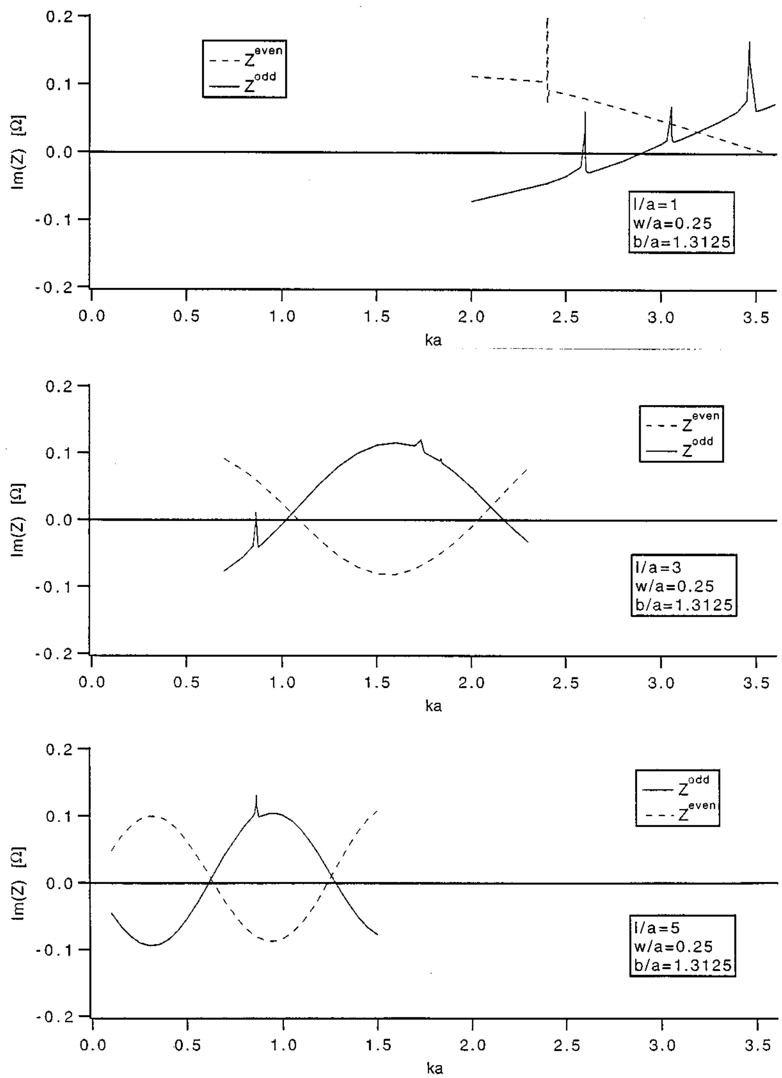

FIG. 7. Even and odd parts of the imaginary part of the impedance for different slot lengths.

$$P=\frac{N_{\rm sl}}{S_b}fq_b^2\kappa, \tag{3.2}$$

where $N_{\rm sl}$ is the total number of slots, S_b is the bunch spacing, $f=c/2\pi R$ is the revolution frequency, and q_b is the bunch charge. The loss factor per slot κ is defined by

$$\kappa=\frac{1}{\pi}\int_0^\infty d\omega\ \mathrm{Re}\ Z(\omega)\exp[-(\omega\sigma/c)^2], \tag{3.3}$$

with σ being the rms bunch length. Using the expression for the real part of the impedance given in Eq. (3.1) we obtain

$$\kappa=\frac{Z_0c\sqrt{\pi}}{256\pi^4a^4\ln(b/a)\sigma^3}(\psi^2+\chi^2). \tag{3.4}$$

The expression above for the loss factor is valid when the bunch spectrum lies below the lowest cutoff of the coaxial structure. The expression given by Eq. (3.4) can then be used for bunch length $\sigma>(b+a)/2$. The available static approximations for ψ and χ, as well as the expressions with the frequency corrections [3,4,8], can be now used to estimate the loss factor for holes of different shape. Expressions identical to Eqs. (3.1) and (3.4) were obtained by Palumbo using a different approach [7].

B. Correction due to the outer wall

It was discussed in detail in [1] that due to the outer wall of the beam pipe the approximate expressions for the impedance in Eq. (3.1) and the loss factor in Eq. (3.4) will be further reduced. This can be taken into account by introducing a b/a correction factor in ψ and χ, as suggested in [1]. This correction factor is on the order of a few percent and therefore can be neglected in general.

C. Correction due to the finite wall thickness

When we consider the wall of finite thickness the separation into inside and outside polarizability and susceptibility is required. The basics of this separation can be found in [3,9]. At low frequencies the dependence of $\psi_{\rm in},\psi_{\rm out},\chi_{\rm in},\chi_{\rm out}$ for the circular and elliptical hole on the wall thickness can be found in [9,10]. At finite frequencies a similar analysis for $\psi_{\rm in},\psi_{\rm out},\chi_{\rm in},\chi_{\rm out}$ of a circular hole can be found in [3,8]. As a

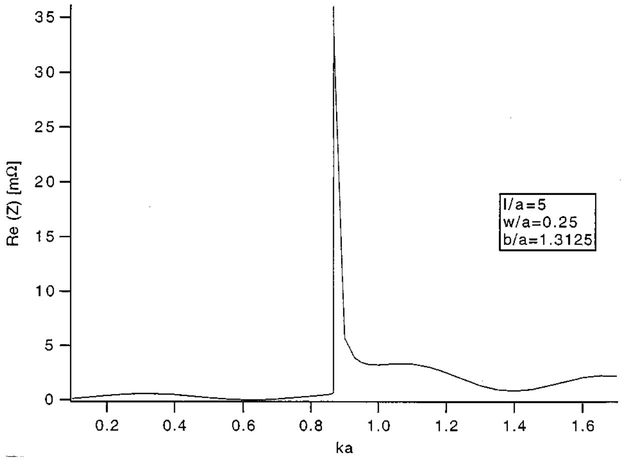

FIG. 8. Frequency dependence of the real part of the impedance for a long rectangular slot ($l/a=5$).

consequence of the analyses mentioned above, the effect of the wall thickness is well understood.

The inside polarizability and susceptibility fall quickly with the wall thickness. The outside polarizability and susceptibility falls much faster with wall thickness than the inside quantities. When we use the developed analytic theory for finite wall thickness, including only the TEM mode, we obtain

$$\mathrm{Re}\left(\frac{Z}{Z_0}\right)=\frac{k^2}{64\pi^3 a^4\ \ln(b/a)}\ (\psi_{\mathrm{out}}^2+\chi_{\mathrm{out}}^2), \tag{3.5}$$

$$\kappa=\frac{Z_0 c\sqrt{\pi}}{256\pi^4 a^4\ \ln(b/a)\sigma^3}\ (\psi_{\mathrm{out}}^2+\chi_{\mathrm{out}}^2). \tag{3.6}$$

Since the expressions in Eqs. (3.5) and (3.6) depend on $\psi_{\mathrm{out}},\chi_{\mathrm{out}}$, instead of ψ,χ in Eqs. (3.1) and (3.4), they will now be greatly reduced due to the wall thickness. As an example, in Table I we present the dependence of $(3/8s^3)^2(\psi_{\mathrm{out}}^2+\chi_{\mathrm{out}}^2)$ on the wall thickness for a small circular hole of radius s, based on numerical calculations [8]. The dependence of χ_{out} and ψ_{out} on t/s for a circular hole (where t is the wall thickness) was studied in a broad frequency range. For low frequencies ($ks \ll 1$) the following asymptotic expressions can be used [9,8]:

$$\ln(3\chi_{\mathrm{out}}/8s^3)\rightarrow -2.405(t/s)-0.886, \tag{3.7}$$

$$\ln(3\psi_{\mathrm{out}}/8s^3)\rightarrow -1.841(t/s)-0.176. \tag{3.8}$$

For low but finite frequencies ($ks \leqslant 1$), the asymptotic constants in the above expressions change. For the electric polarizability χ the asymptotic constant (-0.886) decreases slightly with increasing frequency, as expected from the negative sign of the frequency correction term in the limit of zero wall thickness [8]. For the magnetic susceptibility ψ the asymptotic constant (-0.176) increases with increasing frequency, as expected from the positive sign of the frequency correction term [8], and becomes positive for $ks>0.5$.

TABLE I. Normalized $\chi_{\mathrm{out}}^2+\psi_{\mathrm{out}}^2$ of a circular hole for a different wall thickness (t is the wall thickness and s is the radius of the hole).

t/s	$\left(\frac{3}{8s^3}\right)^2(\psi_{\mathrm{out}}^2+\chi_{\mathrm{out}}^2)$
0.0	1.25
0.01	1.16
0.1	0.71
0.3	0.30
1.0	0.02
3.0	0.000 01

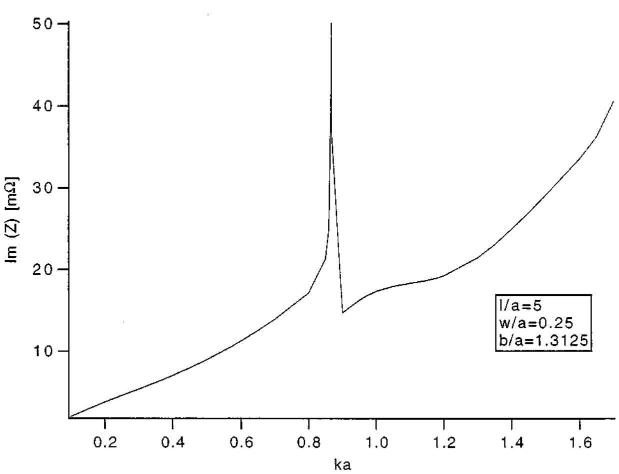

FIG. 9. Frequency dependence of the imaginary part of the impedance for a long rectangular slot ($l/a=5$).

IV. DISCUSSION AND SUMMARY

Investigation of a rectangular slot leads to the following conclusions: At low frequencies the available static approximations for ψ and χ of a rectangle give reasonably good estimates for a relatively long slot. Therefore, Eqs. (2.1) and (3.1) can be used to estimate the imaginary and real parts of the impedance of a relatively long slot at low frequencies. Note that even at low frequencies the static approximation loses its accuracy if the length of the slot is significantly larger than the radius of the pipe. In addition, the static approximation loses its validity at frequencies of the order of $ka=1$.

For the wall of finite thickness we obtain analytically Eqs. (3.5) and (3.6), which should be used to estimate the real part of the impedance and the loss factor of a hole or slot of arbitrary shape. Due to the dependence of these quantities on the outside polarizability and susceptibility the reduction with the wall thickness is very strong.

For a long slot the resonances for the even and odd parts of the imaginary part of the impedance occur near $\lambda=2l$, but due to the strong cancellation between the even and odd parts no resonant behavior is seen for the total imaginary part of the impedance. However, the resonant behavior of the real part of the impedance was observed and should be taken into account if necessary.

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